Test Typed lambda calculi 2006   **October 12 2006**

1.   Let $I \equiv \lambda y.y$, $V \equiv \lambda x.Ixx$, $M \equiv VV$.
        Construct the reduction graph of M, that is find all possible beta-reducts
        of M connected by a directed arc if there is a beta one step reduction
        between them.[Hint. There are 8 nodes and 15 arcs.]

2.   Let $1_3 = (0{\to}0{\to}0{\to}0)$ and $2 = ((0{\to}0){\to}0)$.

        Find all inhabitants in normal form of the types
        $1_3{\to}0{\to}0$ and $2{\to}0{\to}0$.

        You may describe the collections of inhabitants schematically or in words
        or in any other manner, as long as it is clear.
        Remember that a term in normal form has the form
        $\lambda x_1 \ldots x_n.x M_1 \ldots M_m \quad n, m \geq 0$.

3.   Find types for the terms $\lambda xyz.xzzy$, $\lambda xy.x(xIy)$, with $I \equiv \lambda z.z$.