

34. This exercise is on the systems CDV and AO, defined in Figures 15.2 and 15.1.

Check the following equalities

$$\begin{aligned} \llbracket \lambda x.y \rrbracket_{\rho_1}^{\mathcal{F}^{CDV}} &= \emptyset, \\ \llbracket \lambda x.y \rrbracket_{\rho_2}^{\mathcal{F}^{AO}} &= \uparrow (\top \rightarrow \top). \end{aligned}$$

where  $\rho_1(y) = \emptyset$  and  $\rho_2(y) = \uparrow \emptyset$ .

35. Prove Proposition 18.1.5.

36. (i) Let  $\mathcal{T} \in TT$ . By Corollary 18.2.10 we know that  $\mathcal{F}_s^{\mathcal{T}}$  is a  $\lambda I$ -model if it is  $\beta$ -sound. Show that it is not a  $\lambda$ -model by proving

$$\mathcal{F}_s^{\mathcal{T}} \not\models (\lambda x.y)z = y.$$

- (ii) Now let  $\mathcal{T} \in TT^\top$  be  $\beta$ -sound. Show that

$$\mathcal{F}^{\mathcal{T}} \models (\lambda x.y)z = y.$$

without using Theorem 18.2.8 and the corollaries following from it.