30. This exercise is on the type system BCD and the assignment system $\lambda_{\cap\top}^{BCD}$.
(i) Let $\mathcal{E} = \top | C \to D$ where $C \leq D | \mathcal{E} \cap \mathcal{E}$.
Prove

$$(B \in \mathcal{E} \;\&\; B \leq A) \Rightarrow A \in \mathcal{E}.$$

(ii) Let $\top$ not occur in A. Prove

$$\lambda_{\cap\top}^{BCD} \vdash \lambda x.x : A \Leftrightarrow A \equiv (B_1 \to C_1) \cap \ldots \cap (B_k \to C_k)\ , k \geq 1\ , B_i \leq C_i.$$

31. Let $\mathcal{D} \in \mathbf{ALG}$ and $a, a' \in \mathcal{D}$. Prove
(i) a compact $\Rightarrow a \mapsto a'$ is continuous.
(ii) $a \mapsto a'$ is continuous and $a' \neq \bot \Rightarrow a$ is compact.
(iii) a' compact $\Leftrightarrow a \mapsto a'$ compact.

32. Let $\mathcal{D}, \mathcal{D}' \in \mathbf{ALG}$ $b, a_1, \ldots a_n \in \mathcal{D}, b', a_{1'}, \ldots a_{n'} \in \mathcal{D}'$. Prove
$(b \mapsto b') \sqsubseteq (a_1 \mapsto a'_1) \sqcup \ldots \sqcup (a_n \mapsto a'_n) \Leftrightarrow$
$\exists I \subseteq \{1, \ldots, n\}[\sqcup_{i \in I} a_i \sqsubseteq b \;\&\; b' \sqsubseteq \sqcup_{i \in I} a'_i]$.
In $(\Rightarrow)$ we have $I \neq \emptyset$ if $b' \neq \bot_{\mathcal{D}}$.

33. (i) Show by an example that (β-exp) does not hold in $\lambda_{\cap}^{CD}$.
(ii) Verify that the above example is wrong for $\lambda_{\cap\top}^{BCD}$.