

25. Show that **S(KI)(II)** and  $(\lambda x.xxx)\mathbf{S}$  are typable in  $\vdash_{\cap}^{CD}$ .

26. Prove  $\not\vdash_{\cap}^{CD} \lambda xy.xy : \alpha \rightarrow \alpha$ , where  $\alpha$  is any constant.

27. We say that  $M$  and  $M'$  have the same types in  $\Gamma$ , notation  $M \sim_{\Gamma} M'$  if

$$\forall A[\Gamma \vdash M : A \Leftrightarrow \Gamma \vdash M' : A].$$

Prove  $M \sim_{\Gamma} M' \Rightarrow MN \sim_{\Gamma} M'N$ .

28. Prove that Theorem 16.1.10(ii) still holds if the condition of properness is replaced by the following two conditions

$$\begin{aligned} A \leq B &\Rightarrow C \rightarrow A \leq C \rightarrow B. \\ (A \rightarrow B) \cap (C \rightarrow D) &\leq A \cap C \rightarrow B \cap D. \end{aligned}$$

29. Prove

- (i)  $CD$  is not  $\eta$ -sound and  $BCD$  is not  $\eta\top$ -sound.
- (ii)  $\lambda_{\cap}^{CD} \not\vdash (\eta\text{-exp})$  and  $\lambda_{\cap\top}^{BCD} \not\vdash (\eta\text{-exp})$ .