

37. Show that

$$A_1 \rightarrow \dots \rightarrow A_n \rightarrow o \leq_{\beta\eta} A_{\pi_1} \rightarrow \dots \rightarrow A_{\pi_n} \rightarrow o,$$

for any permutation $\pi \in S_n$.

38. Let $A = 3 \rightarrow o \rightarrow o$.

Find by the procedure of the proof of Proposition 3.4.2. a type B such that $\text{rank}(B) \leq 3$ and a $\Phi : A \rightarrow B$ that is injective, modulo $\beta\eta$, on closed terms.

39. Let $A = (2 \rightarrow 2 \rightarrow o) \rightarrow 2 \rightarrow o$ and

$B = (o \rightarrow 1^2 \rightarrow o) \rightarrow 1_2 \rightarrow (o \rightarrow 1 \rightarrow o) \rightarrow o^2 \rightarrow o$. Show that

$$A \leq_{\beta\eta} B.$$

[Hint. Use the term $\lambda z:A \lambda u_1:(o \rightarrow 1^2 \rightarrow o) \lambda u_2:1_2 \lambda u_3:(o \rightarrow 2) \lambda x_1 x_2:o. z[\lambda y_1, y_2:2. u_1 x_1 (\lambda w:o. y_1 (u_2 w)) (\lambda w:o. y_2 (u_2 w))][u_3 x_2].$]