Addenda for the sixth imprinting

The variable convention (keeping the names of bound variables as much different as convenient from those of the free variables) is used throughout the book and is convenient in an informal precise setting. This method was brought to my attention by Thomas Ottmann in 1972 and ever since I used it without refering to him (as it is so natural). After the appearence of this book the convention became baptized with my name. This is unjustified to Ottmann and for this reason I mention explicitly his name in these corrections.

Each row in the list below has generally five items: the page number; the line number skipping to count the lines of pictures (a negative number indicates that one has to count form below); the item to be changed; the sign '$\mapsto$'; the modified item. In some cases that are self-explanatory a different way of indicating the erratum was used.

A wealth of corrections came from Harold Hodes and participants of a seminar directed by Hidetaka Kondoh: Hironobu Kuruma, Jun-ichi Matsuda, Yuuji Nagamatsu, Takanori Nishio and Tetsuo Tanaka. Other corrections were found by to Ingemarie Bethke, Pierre-Louis Curien, Herman Geuvers, Bart Jacobs, Gerard Renardel, Piet Rodenburg and Yiqing Zhu. The more important ones are indicated by the symbol ▶ in the margin.

A few words about progress in theory will be given. In section 6.5 the double fixed-point is stated and proved in two different ways. The first proof is a proof also valid in the λI-calculus. The second proof easily generalizes to the n-fold case. Here we present a third proof, due to Smullyan, that has both virtues.

Theorem (Multiple fixed-point theorem). Given $F_1, \ldots, F_n \in \Lambda$. Then there are $A_1, \ldots, A_n \in \Lambda$ such that

$$\begin{array}{rcl} A_1 & = & F_1 A_1 \ldots A_n \\ & \ldots & \\ A_n & = & F_n A_1 \ldots A_n \end{array}$$

Proof. Given $\vec{F}$, define by the ordinary fixed-point theorem a term A such that

$$A = \lambda f \vec{a}. f(A a_1 \vec{a}) \ldots (A a_n \vec{a}),$$

where $\vec{a} = a_1, \ldots, a_n$. Take $A_i \equiv A F_i \vec{F}$. Then indeed for $1 \leq i \leq n$ one has

$$\begin{array}{rcl} A_i & \equiv & A F_i \vec{F} \\ & = & F_i (A F_1 \vec{F}) \ldots (A F_n \vec{F}) \\ & \equiv & F_i A_1 \ldots A_n. \blacksquare \end{array}$$

A second result is the solution of several hunderd pages in the thesis of Enno Volkerts to problem 21.4.9, see Folkerts [1998].

Theorem. Let F be a closed term considered as a map $\Lambda^\circ/=_{\beta\eta} \to \Lambda^\circ/=_{\beta\eta}$. Then

$$F \text{ is a bijection } \iff F \text{ is } \beta\eta\text{-invertible.}$$

Finally conjecture 17.4.15, concerning the place in the projective hierarchy of the $\boldsymbol{\lambda}$-theory $\boldsymbol{\mathcal{H}\omega}$ axiomatized by equating all unsolvables and the ω-rule, is proved by a complex argument due to Intrigila and Statman [2004].

THEOREM. $\boldsymbol{\mathcal{H}\omega}$ is a Π^1_1-comple $\boldsymbol{\lambda}$-theory.

This settles most open problems of the book. One conjecture that remains open is the range property for $\boldsymbol{\mathcal{H}}$, i.e. the question whether for a closed term F its range modulo equating the unsolvables has cardinality either 1 or $\aleph_0$.

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Errata

	Preface & Contents				
	x	16	ω-rule $\lambda\eta$	$\mapsto$	ω-rule in$\boldsymbol{\lambda\eta}$
	xiv	In diagram	Add solid line from 10 to 14.		
	Chapter 1				
	3	-13	18.5.30	$\mapsto$	18.4.30
	4	-21	Aczel [1980].	$\mapsto$	Aczel [1980]. See also Barendregt et al. [1993] for progress on Curry's program.
		-19	[1980]	$\mapsto$	[1979]
	10	-11	$D = (D', \sqsubseteq')$	$\mapsto$	$D' = (D', \sqsubseteq')$
	13	3	(x, x'_0)	$\mapsto$	$\langle x, x'_0 \rangle$
	15	4	is the least fixed point of f.	$\mapsto$	*is the least fixed point of* f.
		-4	cpo's.	$\mapsto$	cpo's if $f_i(\bot) = \bot$ (strictness).
	16	-11	*retract* of D	$\mapsto$	*retract* of D, notation $X \lhd D$,
	18	1.2.28	*coherent*	$\mapsto$	*coherent* (or *consistently complete*)
		-2	$x{<<}y$	$\mapsto$	$x{<<}y$[1]
	21	1.3.15	coherent algebraic cpo.	$\mapsto$	coherent[2] algebraic cpo.
▶	21	1.3.16(i)	This is incorrect, but holds if each bounded set $Y \subseteq X$ (i.e. $\exists x{\in}X.Y \sqsubseteq x$) has a supremum in X; Jung [1989]		
	Chapter 2				
	24	8	$F(WW) = FX$	$\mapsto$	$F(WW) \equiv FX$
▶	30	5	and the	$\mapsto$	and, if $n > 1$, then the
▶	26	-9	VARIABLE CONVENTION	$\mapsto$	OTTMANN VARIABLE CONVENTION
	35	2	λ	$\mapsto$	$\boldsymbol{\lambda}$
	36	17	*Par abus de language*	$\mapsto$	*Par abus de langage*
	41	15	I	$\mapsto$	I
	46	7	*Applications of **CL** to* $\boldsymbol{\lambda}$	$\mapsto$	*Bases and enumeration*
		-4	$\psi(n)$	$\mapsto$	$\ulcorner\psi(n)\urcorner$
	47	7	M	$\mapsto$	M
	48	-11	Böhm out technique	$\mapsto$	The Böhm out technique
	48	-2	for the	$\mapsto$	in the

[1] This definition is due to Scott [1972]. If D is a continuous lattice, then it is equivalent to

$$x{<<}y \;\Leftrightarrow\; \forall \text{ directed } X \subseteq D.[y \sqsubseteq \bigsqcup X \;\Rightarrow\; \exists z \in X. x \sqsubseteq z],$$

see Gierz et al. [1980], p. 110-111.

[2] The condition of coherence may be dropped.

Chapter 3				
51	2	R	$\mapsto$	$\succ\!\!-$
57	3.1.22(ii)	R-$\infty(M)$	$\mapsto$	R-$\infty(M)$ or $\infty_R(M)$
	3.1.22(iii)	R	$\mapsto$	$\mathbf{R}$
67	2	corollary	$\mapsto$	theorem
	10	Conservation theorem	$\mapsto$	The conservation theorem
71	−3	Sequentiality	$\mapsto$	Sequentiality and stability
72	−4	$\boldsymbol{\beta\eta\Omega}$-reduction	$\mapsto$	$\beta\eta\Omega$-reduction
73	1	Δ-reduction	$\mapsto$	Delta reduction
75	19	$\exists x_1,\ldots,x_n\, x = x_1 \succ\!\!- x_2$	$\mapsto$	$\exists x_0,\ldots,x_n\, x = x_0 \succ\!\!- x_1$
Chapter 4				
83	14	17.2	$\mapsto$	16.2
	−10	16.	$\mapsto$	16.1.
	−1	$\simeq_\eta$	$\mapsto$	$\overline{\sim}_\eta$
84	−4	ω-rule	$\mapsto$	ω-rule in $\boldsymbol{\lambda\eta}$
85	1	Omega incompleteness	$\mapsto$	The ω-rule and $\boldsymbol{\mathcal{H}\eta}$
	−2 (2×)	I	$\mapsto$	$\mathbf{I}$
Chapter 5				
86	11	Plotkins	$\mapsto$	Plotkin's
87	17	Scott [1980]	$\mapsto$	Scott [1980a]
90	14	$(\lambda^* x, Q)$	$\mapsto$	$(\lambda^* x.Q)$, otherwise.
91	6	$\mathfrak{M}_1$	$\mapsto$	(iv) $\mathfrak{M}_1$
	8	(iv)	$\mapsto$	(v)
	5.1.14 all over } 5×	$[\![\ ,\ ]\!]$	$\mapsto$	$(\ ,\ )$ ⟨respectively⟩
92	−5	$[\![\ ,\ ]\!]$	$\mapsto$	$(\ ,\ )$ ⟨respectively⟩
	−1	$]\!]^{\mathfrak{A}}$	$\mapsto$	$]\!]^{\mathfrak{M}}$
93	(6×)	$\lambda \vdash$	$\mapsto$	$\boldsymbol{\lambda} \vdash$
	−4	$\lambda \vdash A_\lambda = B_\lambda \ \Rightarrow\ \mathfrak{M} \models A_{\lambda,CL} = B_{\lambda,CL}$, by 1,	$\mapsto$	
		$\lambda \vdash A_\lambda = B_\lambda \ \Rightarrow\ \mathfrak{M} \models A_\lambda = B_\lambda$, by 1, $\Rightarrow\ \mathfrak{M} \models A_{\lambda,CL} = B_{\lambda,CL}$, by definition of $[\![A_\lambda]\!]^{\mathfrak{M}}_\rho$		
	−1	$\Lambda(\mathfrak{M})$	$\mapsto$	$\Lambda(\mathfrak{M}_1)$
94	−4	5.5.8	$\mapsto$	5.6.8
	−1	⟨not exactly⟩ $\mathcal{M} \models$	$\mapsto$	$\mathfrak{M} \models$
	−1 (2×)	λ	$\mapsto$	λ^*
95	1	Meyer [1980]	$\mapsto$	Meyer [1982]
	2	Scott [1980]	$\mapsto$	Scott [1980a]
	5.2.8 (11×)	λ	$\mapsto$	λ^*
	5.2.9 (8×)	λ	$\mapsto$	λ^*
	5.2.10 (4×)	λ	$\mapsto$	λ^*

Chapter 5 (continued)				
96	all over (3×)	$\boldsymbol{K}, \boldsymbol{S}$	$\mapsto$	K, S, ⟨respectively⟩
	−11, −10, −9	λ	$\mapsto$	$\boldsymbol{\lambda}$
97	1	$[\![\ ,\ ]\!]$	$\mapsto$	$(\ ,\ )$ ⟨respectively⟩
	−5	Jacopini [1975a]	$\mapsto$	Jacopini [1975]
	−4, −3, −2	$\boldsymbol{K}$	$\mapsto$	K
	−4, −3, −2, −1	$\boldsymbol{S}$	$\mapsto$	S
98	4	$\boldsymbol{K}$	$\mapsto$	K
		$\boldsymbol{S}$	$\mapsto$	S
99	8	$\boldsymbol{S}yxz$	$\mapsto$	$\boldsymbol{S}xyz$
►	13	$(\lambda)_c \vdash M = N \Leftrightarrow \lambda \vdash M = N$	$\mapsto$	$(\boldsymbol{\lambda})_c \vdash A = B \Leftrightarrow \boldsymbol{\lambda} \vdash A_\lambda = B_\lambda$
	−1	$\boldsymbol{R}$-axiom.	$\mapsto$	$\boldsymbol{R}$-ax.
100	11	$\in \lambda$	$\mapsto$	$\in \Lambda$
	15	ξ-ax	$\mapsto$	$\boldsymbol{\xi}$-ax
	−2	ξ-ax	$\mapsto$	$\boldsymbol{\xi}$-ax
	−2, −1	-*rule*	$\mapsto$	-rule
101	2	By corollary 5.2.23.	$\mapsto$	Left to the reader.
	3 − 6 (4×)	ω	$\mapsto$	$\boldsymbol{\omega}$
	6	theorem 4.1.15(i).	$\mapsto$	proposition 4.1.15(i).
	8	-*rule*	$\mapsto$	-rule
	10	$\mathfrak{T} \models \boldsymbol{R}$	$\mapsto$	$\mathfrak{T} \vdash \boldsymbol{R}$
	11	5.2.12(ii)	$\mapsto$	5.2.12(iii)
103	−11	$(x := [\![N]\!]_\rho)$	$\mapsto$	$(x := [\![z]\!]_{\rho(z:=[\![N]\!]_\rho)})$
104	4	$\Lambda(\mathfrak{M})$	$\mapsto$	$\Lambda(\mathfrak{M}_1)$
105	5	$x.y$	$\mapsto$	$x \cdot y$
	14	$[\![D]\!]$	$\mapsto$	$[\![P]\!]$
107	−13	5.7.7.	$\mapsto$	5.8.7.
		18.5.29	$\mapsto$	18.4.29
		18.4.31	$\mapsto$	20.6.22
	−7	categorial	$\mapsto$	categorical
108	5	$p_2)$	$\mapsto$	$p_2\rangle$
	7	$A, B \in C$	$\mapsto$	$A, B \in \mathbb{C}$
	14	⟨Delete⟩ It then follows that the same holds for all $f, g : A \to B$.		
109	−11	λ	$\mapsto$	$\lambda\!\!\lambda$
110	8	by 4.1.(1)	$\mapsto$	by the note after 5.5.1.(ii)
111	8, 9	⟨Delete two lines.⟩		
	10	$\Rightarrow [\![P]\!]_{\Delta,x} = [\![Q]\!]_{\Delta,x}$	$\mapsto$	$[\![P]\!]_{\Delta,x} = [\![Q]\!]_{\Delta,x}$, by the IH,
	13	$\mathfrak{M}(C)$	$\mapsto$	$\mathfrak{M}(\mathbb{C})$
	−5, −4 (2×)	$\Rightarrow$	$\mapsto$	$\Rightarrow\ \forall x \in \mathfrak{M}$

Chapter 5 (continued)				
112	6	$F \circ G = \mathrm{id},$	$\mapsto$	$F \circ G = \mathrm{id},\ G$ is mono,
	8	g	$\mapsto$	$g,$ since p_2 is epi.
113	5.5.10	M	$\mapsto$	P
		N	$\mapsto$	Q
114	−16	1	$\mapsto$	**1**
	−8	Scott [1980]	$\mapsto$	Scott [1980a]
115	5.5.7 until end of 5.6, except 5.6.3	$\boldsymbol{I}, \boldsymbol{K}, \boldsymbol{S}$	$\mapsto$	$\mathsf{I}, \mathsf{K}, \mathsf{S}$ ⟨respectively⟩
	−5	$t \to \boldsymbol{I}$	$\mapsto$	$T \to \boldsymbol{I}$
117	9	Scott [1980]	$\mapsto$	Scott [1980a]
	−10	1	$\mapsto$	**1**
	5.6.3	λ	$\mapsto$	λ^*
118	5.6.3	λ	$\mapsto$	λ^*
	14	1	$\mapsto$	**1**
	16	$\mathbf{K1}_n$	$\mapsto$	$\mathbf{K1}_p$
	17	$\mathbf{1}_n$	$\mapsto$	$\mathbf{1}_p$
	−4	λ	$\mapsto$	λ^*
120	13	1	$\mapsto$	**1**
124	14	$(X, \cdot, \lambda)$	$\mapsto$	$(X, \cdot, \sqsubseteq)$
	−1	$\boldsymbol{B}$	$\mapsto$	$\mathcal{B}$
125	−12	is	$\mapsto$	vs.
126	5	$M(\mathcal{T})$	$\mapsto$	$\mathfrak{M}(\mathcal{T})$
	18	$\mathcal{B}$	$\mapsto$	$\mathfrak{B}$
127	18	**21.4 Exercises**	$\mapsto$	*21.4 Exercises*
	−7	$\mathcal{F} = (X, \cdot\,)$	$\mapsto$	$\mathfrak{M} = (X, \cdot\,)$
128	15, 16	$\subsetneq$	$\mapsto$	$\sqsubsetneq$
	21	$\mathbf{I}, \mathbf{K}, \mathbf{S}, \boldsymbol{\Omega}$	$\mapsto$	$\mathsf{I}, \mathsf{K}, \mathsf{S}, \Omega$
Chapter 6				
150	−16	2.3.5	$\mapsto$	2.4.5

	Page	Line	Old		New
			Chapter 7		
	152	18	$CL \vdash$	$\mapsto$	$\boldsymbol{CL} \vdash$
		−8	$(\lambda^* x.Q).$	$\mapsto$	$(\lambda^* x.Q),\quad$ otherwise.
	153	−8	Suppose $\vec{x} \notin \mathrm{FV}(\vec{Q})$. Then	$\mapsto$	Then
	155	2	remark 3.1.7	$\mapsto$	definition 3.1.5
		8	$\boldsymbol{\lambda}$-term	$\mapsto$	λ-term
		§7.2	M, N, L	$\mapsto$	P, Q, R $\langle$respectively$\rangle$
	156	−1	$\lambda \vdash$	$\mapsto$	$\boldsymbol{\lambda} \vdash$
	157	2	$\lambda \vdash$	$\mapsto$	$\boldsymbol{\lambda} \vdash$
	161	11	$= \boldsymbol{S(KK)(S(S(KS)(S(KK)(SKK)))(K(SKK)))}$	$\mapsto$	$\boldsymbol{S(KK)(S(S(KS)K)(K(SKK)))}$
►		−1	$\lambda x.\mathsf{II} \to_\beta \lambda x.\mathsf{I}$	$\mapsto$	$\lambda x.\mathsf{I}x \to_\beta \mathsf{I}$
►	162	2	$\boldsymbol{S(KI)(KI)} \not\twoheadrightarrow_w \boldsymbol{KI}$	$\mapsto$	$\boldsymbol{S(KI)I} \not\twoheadrightarrow_w \boldsymbol{I}$
	162	8	w-solvable	$\mapsto$	*w-solvable*
		−5	$S's$	$\mapsto$	$\boldsymbol{S}'s$
	163	4	I	$\mapsto$	$\boldsymbol{I}$
		16	K, S	$\mapsto$	$\boldsymbol{K}, \boldsymbol{S}$ $\langle$respectively$\rangle$
		−7	$\overset{!}{\twoheadrightarrow}$	$\mapsto$	$\overset{!}{\to}$
		−5	K, S	$\mapsto$	$\boldsymbol{K}, \boldsymbol{S}$ $\langle$respectively$\rangle$
			Chapter 8		
	162	−2	Petorossi	$\mapsto$	Pettorossi
	167 – 168	(6×)	#	$\mapsto$	$\sharp$
	169	−9	Proof.	$\mapsto$	Proof. (i)
	171	13	$x \notin F$	$\mapsto$	$x \notin \mathrm{FV}(F)$
	173	9	$N_1 N_2 \ldots N_n$	$\mapsto$	$N_1 N_2 \ldots N_k$
	177	2	$\vec{x}, \vec{w}$	$\mapsto$	$\vec{x}\vec{w}$
	178	7	$M_{CL} N_{CL}$	$\mapsto$	$M_{CL} \vec{N}_{CL}$
	179	13	one has ψ	$\mapsto$	one has ϕ
	180	11	occurrences if redexes	$\mapsto$	occurrences of redexes
	181	2	$M = M_0$	$\mapsto$	$M \equiv M_0$
		10	$n \in N$	$\mapsto$	$n \in \mathbb{N}$
	(3×)	14, 15, 19	$=$	$\mapsto$	$\equiv$
	182	−8	m is solvable	$\mapsto$	$\ulcorner m \urcorner$ is solvable
	184	11	KH_4	$\mapsto$	$\mathsf{K}H_4$
		15	$\mathsf{K}^3\mathsf{I}$	$\mapsto$	$\mathsf{K}^4\mathsf{I}$
		−4	16.3.15	$\mapsto$	17.3.15
		−2	S	$\mapsto$	S

	Chapter 9				
	186	14	$M \in \Lambda$	$\mapsto$	$\lambda x.M \in \Lambda$
		$-5, -4$	I	$\mapsto$	I
	193	-8	N_m	$\mapsto$	N_n
	198	-2 (2×)	$\#$	$\mapsto$	$\sharp$
►	201	-2	$\forall k {\leq} \mathrm{lh}(\alpha)\ \alpha(2k) {\leq} m$	$\mapsto$	$\forall k {\leq} n\ \alpha(2k) {\leq} m$
	203	19	$k = m + k_0$	$\mapsto$	$k = m + k_0 + 1$
		-8	$M\mathsf{I}^{\sim p}$	$\mapsto$	$M\mathsf{I}^{\sim p+1}$
		-8	$\in$	$\mapsto$	$\in_\beta$
	205	-16	$P_{n+1}, \ldots P_p$	$\mapsto$	$P_{n+1} \ldots P_p$
	208	2	$\supset$	$\mapsto$	$\vec{\supset}$
	209	-6	9.1.7	$\mapsto$	9.1.6
	211	14, 16 (2×)	P_p	$\mapsto$	$P_{p'}$
	Chapter 10				
	216	-6	Böhm tree	$\mapsto$	*Böhm tree*
	220	$-4, -3$ (2×)	$\uparrow$	$\mapsto$	$= \perp$
	222	20	$i > m$	$\mapsto$	$i \geq m$
		-3	$M_{\langle 0\rangle} = \mathbf{\Omega}$	$\mapsto$	$M_{\langle 0\rangle} \equiv \mathbf{\Omega}$
	223	-3	$\uparrow$ else.	$\mapsto$	$\perp$ if $\mathrm{lh}(\alpha) = k$ and $\alpha \in A$; $\uparrow\uparrow$ else.
	226	15	$\#B_\alpha$	$\mapsto$	$\sharp B_\alpha$
	227	-9	$\vec{x}_\alpha, y_\alpha$	$\mapsto$	$\vec{x}_\alpha, y_\alpha, m_\alpha$
	228	1	THEOREM.	$\mapsto$	THEOREM (Bergstra and Klop [1980]).
		-12	$\ulcorner \alpha * \langle 0\rangle \urcorner$	$\mapsto$	$\llcorner \alpha * \langle 0\rangle \lrcorner$
		-11	$\#B_\alpha$	$\mapsto$	$\sharp B_\alpha$
►		-3	$\forall \alpha\ \{\vec{x}_\alpha\} \subseteq \bigcup\{\mathrm{FV}_A(\beta) \mid \beta > \alpha\}$	$\mapsto$	$\forall \alpha\ [A(\alpha)\ \Rightarrow\ \{\vec{x}_\alpha\} \subseteq \{y_\alpha\} \cup \bigcup\{\mathrm{FV}(A_{\alpha * \langle i\rangle}) \mid 0 \leq i \leq m_\alpha\}]$.
	229	-5	PROOF.	$\mapsto$	PROOF. (i)
	230	6	*tree* topology	$\mapsto$	*tree topology*
		7	$BT : \Lambda \to \mathfrak{B}$	$\mapsto$	$\mathrm{BT} : \Lambda \to \mathfrak{B}$
		-9 (3×)	BT	$\mapsto$	BT
	231	-5	$\lambda x.x$	$\mapsto$	$\langle \lambda x.x, 0\rangle$
	232	10.2.9	BT	$\mapsto$	BT
	233	1	$A : X$	$\mapsto$	$A; X$
►	236	-5	$x_n \leq_\eta A_m$	$\mapsto$	$x_n \leq_\eta A_m\ \&\ x_n \not\equiv y$
►	237	-4	$(M; X_\alpha)_\alpha$	$\mapsto$	$M(\mathrm{BT}(M); X_\alpha)_\alpha$
	238	-11	Σ_1	$\mapsto$	$\Sigma_1 \times \mathbb{N}$
	245 – 246	(3×)	BT	$\mapsto$	BT
	247	5	$(\lambda \vec{x}.M)\vec{N}^*$	$\mapsto$	$(\lambda y\vec{x}.M)P\vec{N}^*$

	Chapter 10 (continued)				
	248	2	$= \mathsf{P}_2\mathsf{I}(\mathsf{P}_2\mathsf{I}y\boldsymbol{\Omega})$	$\mapsto$	
	249	11	U^n_j	$\mapsto$	U^m_j
		13	$xM_1 \dots M_m$	$\mapsto$	$xM_0 \dots M_m$
	250	$-5, -4$ (2×)	$\vec{P}$	$\mapsto$	$\vec{R}$
		-2	(1)	$\mapsto$	(i)
	251	9	$M\mid\beta = N\mid\beta$	$\mapsto$	$M\mid\beta \sim N\mid\beta$
		18	$\lambda\vec{x}.yN_{i1} \dots N_{im}$	$\mapsto$	$\lambda\vec{x}.yM_{i1} \dots M_{im}$
		-6	10.3.6 (ii).	$\mapsto$	10.3.6 (ii), that also holds for virtual nodes.
	253, 254	$-13, 5$	along	$\mapsto$	up to
	254	-6	(i) By	$\mapsto$	(i) It suffices to show this for closed P, Q. By
	256	4	$\simeq_\eta$	$\mapsto$	$\eqsim_\eta$
		7	along	$\mapsto$	up to
	257	3	$\mathcal{F}$ if	$\mapsto$	$\mathcal{F}$ is
		4	10.2.31	$\mapsto$	10.2.13
	258	$-9, -6$	P_4	$\mapsto$	P_4
	261	2	$\lambda\eta \vdash$	$\mapsto$	$\boldsymbol{\lambda\eta} \vdash$
		-3	(i)	$\mapsto$	
	263	-8	$xM^k_{11} \dots\dots\dots M^k_{1m_q}$	$\mapsto$	$xM^q_{11} \dots\dots\dots M^q_{1m_q}$
	265	$10 - 21$ (19×)	π	$\mapsto$	π_1
		$11, 13$ (2×)	$k =$	$\mapsto$	$p =$
		13	$\lambda\vec{y}_i.z_1$	$\mapsto$	$\lambda\vec{y}_1.z_1$
		16	$k >$	$\mapsto$	$p >$
		$21, 22$ (2×)	Max	$\mapsto$	min
▶	265	26	Before 10.5.21 add the following definition. 10.5.20A. Definition. (i) π is called $\mathcal{F}$-*nonconfusing* $\Longleftrightarrow$ $\forall M, N \in \mathcal{F}.[M \sim N \;\Leftrightarrow\; M^\pi \sim N^\pi]$ & $[M\mid\langle\,\rangle\downarrow \;\Leftrightarrow\; M^\pi$ solvable] (ii) $\mathcal{F}^o_I = \{\pi \in I \mid \pi$ is $\mathcal{F}$-nonconfusing and M^π is solvable$\}$.		
	266	2	b_p	$\mapsto$	b_q
		$-12, 11, -9$	b_p	$\mapsto$	b_q
▶	267	3	$\mathcal{F}$-faithful	$\mapsto$	$\mathcal{F}$-nonconfusing
▶		$8, 14, 15, 18,$ $-6, -1$ (6×)	$\mathcal{F}^*_I$	$\mapsto$	$\mathcal{F}^o_I$
▶	268	2	$\mathcal{F}^*_I$	$\mapsto$	$\mathcal{F}^o_I$
	272	15	$x \in \mathrm{BT}(Fx)$	$\mapsto$	$x \in \mathrm{BT}(Fx)$, i.e. $x \in \mathrm{FV}(\mathrm{BT}(Fx))$,
		16	$\lambda \vdash$	$\mapsto$	$\boldsymbol{\lambda} \vdash$
		15	⟨something like⟩ $\Lambda_{\mathsf{I}}\mathcal{B}$	$\mapsto$	$\Lambda_{\mathsf{I}}\mathfrak{B}$

Chapter 11				
279	2	$\ldots(P_2[x_2 := (\ldots\Delta'_1\ldots)])$	$\mapsto$	$\ldots(P_2[x_2 := (\ldots\Delta'_1\ldots)])\ldots$
282	2-nd diagram	Arrow from `M~` to `N~` should have as label β' (not β).		
285	5	$\{\Delta \mid \Delta \in P$	$\mapsto$	$\{\lvert\Delta\rvert \mid \Delta \in P$
287	-11	an *weighting*	$\mapsto$	a *weighting*
288	-1	274	$\mapsto$	278
289	-7	$\lambda_i x_2.P_0$	$\mapsto$	$\lambda_j x_2.P_0$
291	4	$\{M \mid M \underset{\text{dev}}{\twoheadrightarrow} N\}$	$\mapsto$	$\{N \mid M \underset{\text{dev}}{\twoheadrightarrow} N\}$
	-4	M	$\mapsto$	$(M, \mathcal{F})$
292	-11	$\omega = \lambda x.xx$	$\mapsto$	$\omega \equiv \lambda x.xx$
	-4	$\underset{1}{\rightarrow}$	$\mapsto$	$\underset{1}{\twoheadrightarrow}$
294	5	$M' \rightarrow_{\beta_0} N'$	$\mapsto$	$M' \twoheadrightarrow_{\beta_0} N'$
296	-3	$\sigma : M \rightarrow N$	$\mapsto$	$\sigma : M \twoheadrightarrow N$
298	-3	$M' \underset{1,\text{i}}{\twoheadrightarrow} M$ even	$\mapsto$	even $M' \underset{1,\text{i}}{\twoheadrightarrow} N$
300	$-12 - -4$	Argument can be simplified by distinguishing $N \equiv \lambda x.N_0$ and $N \equiv N_0 N_1$.		
	-2 $(2\times)$	$\vec{M}$	$\mapsto$	$\vec{P}$
	-1	$\vec{N}$	$\mapsto$	$\vec{Q}$
	-1	$M_i \twoheadrightarrow N_i$	$\mapsto$	$P_i \twoheadrightarrow Q_i$
301	$2, 4, 7$ $(3\times)$	$\vec{N}$	$\mapsto$	$\vec{Q}$
Chapter 12				
302	$6, 7$ $(2\times)$	$\rightarrow$	$\mapsto$	$\twoheadrightarrow$
303	9	12.1.1	$\mapsto$	12.1.1A
303	16	$\boldsymbol{R}$-=-reduction	$\mapsto$	R-=-reduction
304	2	$\stackrel{\smile}{=}$	$\mapsto$	$\cong$
305	7	D_4	$\mapsto$	D_3
	1st diagram	$\sigma\ \rho,\ \rho\ \sigma$	$\mapsto$	$\sigma/\rho,\ \rho/\sigma$ $\langle$respectively$\rangle$
310	-7	Since by (2)	$\mapsto$	Since by (3)
316	-4	$\cong$ an	$\mapsto$	$\cong$ is an
319	-1	Δ_{n-1}	$\mapsto$	Δ_{n+1}
320, 321	1, 5	lemma 2.1.12	$\mapsto$	proposition 2.1.12
322	1	(σ)	$\mapsto$	(σ_k)

	Chapter 13				
	324	-7	effective	$\mapsto$	effective[3]
	325	-7	β	$\mapsto$	$\boldsymbol{\beta}$
	327	-3	$=$	$\mapsto$	$\equiv$
	328	-5	β	$\mapsto$	$\boldsymbol{\beta}$
	329	Fig. 13.3.	$\mathtt{N}'$	$\mapsto$	$\mathtt{N}$
	330	3	$=$	$\mapsto$	$\equiv$
	331	1	reduction	$\mapsto$	*reduction*
		-4	M_1	$\mapsto$	M
		Fig. 13.6	The gk-arrows should be dotted.		
	332	7	Proof.	$\mapsto$	Proof. (i)
	334	$4, 5$ ($2\times$)	$\mathcal{C}$	$\mapsto$	$\mathcal{C}_1$
►		4	$\twoheadrightarrow$	$\mapsto$	$=$
	338	-1	$0 \le i \le n$	$\mapsto$	$0 \le i < n$
	340	-11 ($3\times$)	$\to$	$\mapsto$	$\twoheadrightarrow$
	342	6	order i	$\mapsto$	order $i-1$
	343	11	13.4.11 (i)	$\mapsto$	13.4.11 (ii)
		15	13.4.11 (ii)	$\mapsto$	13.4.11 (i)
		Fig. 13.9 ($5\times$)	Arrows labelled ‘`cpl`’ should have double heads.		
	344	1	theorem 11.2.20	$\mapsto$	proposition 11.2.20
		11	Bergstra-Klop [198+]	$\mapsto$	Bergstra-Klop [1982]
►	345	$-6, -4$ ($3\times$)	Replace $\|\|M\|\|, \|\|N\|\|$ by $\|\|M\|\|_1, \|\|N\|\|_1$, respectively, where $\|\|..\|\|_1$ is defined in the proof of 13.3.5.		
►		-1	in $\Lambda_{\|\|M\|\|}$.	$\mapsto$	in $\Lambda_{\|\|M\|\|_1}$ and does not contain an F-cycle.
►	346	$1, 2$	Between ‘otherwise’ and ‘F’ insert: ‘at least one of the following two situations holds: 1. the F-path of M contains an F-cycle; 2. the F-path of M does not lie within $\Lambda_{\|\|M\|\|_1}$. Case 1 is impossible as F is a B-optimal normalizing strategy. If case 2 holds, then’		
		-9	AB	$\mapsto$	AB
	347	5	I	$\mapsto$	I
		9	l-1-optimal	$\mapsto$	L-1-optimal
		-3	$B(\mathsf{II})) =$	$\mapsto$	$B(\mathsf{II}))) =$

[3] Some readers prefer the word ‘efficient’.

Chapter 14				
348	5 (2×)	$\#$	$\mapsto$	$\sharp$
	11 (2×)	$\#$	$\mapsto$	$\sharp$
350	Fig. 13.10	Figure c should be upside down.		
351	-2	than	$\mapsto$	then
355	17	$\rightarrow_{lab.\beta}$	$\mapsto$	$\rightarrow_{\text{lab.}\beta}$
356	15	$L_1 = xV_1 \dots V_M$	$\mapsto$	$L_1 \equiv xV_1 \dots V_M$
	-5	$M = M_1M_2$	$\mapsto$	$M \equiv M_1M_2$
358	7	$(x\vec{N})^* \equiv \perp\vec{N}$	$\mapsto$	$(x\vec{N})^* \equiv \perp\vec{N}^*$
360	4	an alternative proof	$\mapsto$	a proof
361	-9	$k = 0$	$\mapsto$	$k \in \{0, 1\}$
	-9	$k > 0$	$\mapsto$	$k > 1$
362	-5	14.2.8	$\mapsto$	14.2.7
363	2	theorem	$\mapsto$	proposition
	5, 6	elementary and diagram	$\mapsto$	elementary diagram
	-11	Δ'_j	$\mapsto$	Δ'_i
Chapter 15				
364	-7	$\boldsymbol{\beta}(\perp)$	$\mapsto$	$\boldsymbol{\beta}\perp$
369	8	$P \equiv D[\perp, \dots, \perp]$	$\mapsto$	$P \;_{\beta}\!\twoheadleftarrow D[\perp, \dots, \perp]$
	10	$C[\vec{P}] \equiv C[D[\vec{\perp}]]$	$\mapsto$	$C[\vec{P}] \;_{\beta}\!\twoheadleftarrow C[D[\vec{\perp}]]$
371	15	lemma 14.3.15	$\mapsto$	lemma 14.3.14
372	8, 10 (2×)	$C[M] = N$	$\mapsto$	$C[M] \equiv N$
373 – 374	(9×)	Replace superscripts $^{(n)}$ and $^{(m)}$ by $^{[n]}$, $^{[m]}$, respectively.		
382	16	(i) Suppose	$\mapsto$	Suppose
	16	redex. Show	$\mapsto$	redex. (i) Show
	-8	$M \in A$	$\mapsto$	$M \in \Lambda$
385	10 (2×)	$\equiv$	$\mapsto$	$=$
	-5	(3) and (1)	$\mapsto$	(2) and (1)
	-2	$(xP_1 \dots P_n)^\eta$	$\mapsto$	$(\dots(x^{\cdots}P_1)^{\cdots} \dots P_n)^\eta$
386	-6	redexes	$\mapsto$	β-redexes
387	-4	$\lambda x_1 \cdots x_n.x_1N_2 \dots N_n$	$\mapsto$	$\lambda x_n \cdots x_1.x_1N_2 \dots N_n$
388	6	$NL_1 \dots L_n$	$\mapsto$	$NL_n \dots L_1$
	7	$y_2P_{11} \dots P_{1k_1}$	$\mapsto$	$y_2P_{21} \dots P_{2k_2}$
	-9	$M \not\equiv \boldsymbol{\Omega}$	$\mapsto$	$M \not\equiv \Omega$
	12 – 19 (5×)	Ω	$\mapsto$	$\boldsymbol{\Omega}$
390	-8	$\downarrow\beta$	$\mapsto$	$\downarrow_\beta$
393	5	$((\lambda x.z(xx))\omega_3)$	$\mapsto$	$((\lambda x.z(xx))\omega_3))$
395	-10	15.2.4(ii)	$\mapsto$	15.2.4(iii)

	Page	Line	Old		New
			Chapter 15 (continued)		
	396	Fig. 15.4	Interchange labels (1) and (2).		
	398	1	$\lambda y, M_0$	$\mapsto$	$\lambda y.M_0$
		4	$H \subseteq M_i$	$\mapsto$	$H \subset M_i$
		−17	$\lambda x_m \cdot y$	$\mapsto$	$\lambda x_m.y$
	400	−10	δ_C	$\mapsto$	$\boldsymbol{\delta}_C$
	401	6	$\Lambda\boldsymbol{\delta}$	$\mapsto$	$\Lambda^\circ\boldsymbol{\delta}$
	403	−3	$M_1, \ldots, M_n$	$\mapsto$	$M_1 \ldots M_n$
		−2	$\mathrm{BT}(M_1) \quad \mathrm{BT}(M_n)$	$\mapsto$	$\mathrm{BT}(M_1) \ldots \mathrm{BT}(M_n)$
	406	diagram	Left arrow with β should be doubly headed.		
			Chapter 16		
	413	−10	$M \approx N$	$\mapsto$	$M' \approx N'$
	416	−10	19.2.12	$\mapsto$	19.2.9
	418	−7	19.2.12	$\mapsto$	19.2.9
	419	−11	π_n	$\mapsto$	$\boldsymbol{\pi}_n$
	420	−12	$M \not\equiv N'$	$\mapsto$	$M' \not\equiv N'$
		−3	$\simeq_\eta$	$\mapsto$	$\overline{\sim}_\eta$
	422	2	$\boldsymbol{\Theta}$	$\mapsto$	Y
		2	$B \twoheadrightarrow$	$\mapsto$	$Bx \twoheadrightarrow$
	425	−10, −9 (4×)	BT	$\mapsto$	BT
	426	in fig. (2×)	BT	$\mapsto$	BT
	427	3	ω	$\mapsto$	$\boldsymbol{\omega}$
	429	2	14.4.5	$\mapsto$	15.1.5
		14	$\boldsymbol{\Theta}(jxy.x(jy))$	$\mapsto$	$\boldsymbol{\Theta}(\lambda jxy.x(jy))$
	429	17	$\sqsupseteq$	$\mapsto$	$\underset{\sim}{\sqsupset}$
		−8	$\forall z{\in}\Lambda^\circ$	$\mapsto$	$\forall Z{\in}\Lambda^\circ$
		−5	$AZ = AZ$	$\mapsto$	$AZ = AZ'$
		−4	$AZ = A'Z$	$\mapsto$	$AZ = AZ'$
	430	9	$M \sqsubseteq N$	$\mapsto$	$M \underset{\sim}{\sqsubset} N$
▶		−4	$J = \boldsymbol{\lambda} + \mathsf{I} = \boldsymbol{\Omega}_3 \equiv \omega_3\omega_3$	$\mapsto$	$J = \boldsymbol{\lambda} + \mathsf{I} = \Omega$ and $\Omega_3 \equiv \omega_3\omega_3$
		−3	$=_{\mathsf{J}}$	$\mapsto$	$=_J$
			Chapter 17		
▶	435	−9, −10	$O_M = \{N \mid FM \in O\}$	$\mapsto$	$O_M = \{N \mid FN \in O\}$
	443	4	15.1.9	$\mapsto$	16.1.9(ii)
	465	11	17.1.9 (ii)	$\mapsto$	17.1.9

Page	Line	Old		New
		Chapter 18		
468	4	Palamidessi Catuscia	$\mapsto$	Catuscia Palamidessi
473	-6	$\lambda^G x.A$	$\mapsto$	$\lambda x.A$
▶ 476	12	$(\lambda x.A)\emptyset = \emptyset$	$\mapsto$	$(\lambda x.A)\emptyset \neq \emptyset$
	-2	$e_k \not\subseteq e_k$	$\mapsto$	$e_k \not\subseteq e_{k'}$
477	$-2, -1$ (2×)	$\psi(x)$	$\mapsto$	$\psi(x')$
481	12	$\langle \sqcup_n x_{\min}(n,i)\rangle i \in \mathbb{N}$	$\mapsto$	$\langle \sqcup_n x_{\min}(n,i)\rangle_{i\in\mathbb{N}}$
483	11	$y \in {}_n$	$\mapsto$	$y \in D_n$
485	-3	$\Phi_{n,\infty}(\lambda\!\!\lambda y \in D_n$	$\mapsto$	$\Phi_{n+1,\infty}(\lambda\!\!\lambda y \in D_n$
491	18.4.1	$P\omega \forall \mathbf{1} = \mathsf{I}$	$\mapsto$	$P\omega \not\models \mathbf{1} = \mathsf{I}$
492	-10	15.3.4	$\mapsto$	15.4.4
		Chapter 19		
508	1	19.2.14	$\mapsto$	19.2.11
509	-6	operators	$\mapsto$	combinators
	-5	combinator	$\mapsto$	operator
		Chapter 20		
513		5.3.25	$\mapsto$	5.2.23(i)
518	12	Wadsworth.	$\mapsto$	Wadsworth. Write $x \in \mathrm{BT}(M)$ for $x \in \mathrm{FV}(\mathrm{BT}(M))$.
521	-5	19.3.15	$\mapsto$	18.3.15
		Appendices		
570	11	[198+]	$\mapsto$	[1982]
576	-3	[1980]	$\mapsto$	[1979]
581	6 (2×)	v_2	$\mapsto$	v_1
584	20	[198-]	$\mapsto$	[198?]
		Addenda		
582	6	$xz(yx)$	$\mapsto$	$xz(yz)$

References				
585	-4	[198-]	$\mapsto$	[1983]
	-3	to appear.	$\mapsto$	48, pp. 931-940
586	-22	[1982]	$\mapsto$	[1982a]
	-11	Add: Bezem, M. A.		
		[1985] Isomorphisms between HEO and HRO^E, ECF and ICF^E. *Journal of Symbolic Logic*, **50**, pp. 359-371.		
	-8	[1980]	$\mapsto$	[1979]
589	15 ($2\times$)	[1980]	$\mapsto$	[1979]
595	$5, 6$	(to appear).	$\mapsto$	no. 3, pp. 271–286, 287–302, 303–325.
597	-9	$\mathfrak{p}$-functions	$\mapsto$	$\mathfrak{p}$-function
598	-8	AGM	$\mapsto$	ACM
Index of Names				
599	$R17$	Add: Bezem, M. [1985] 566		
	$L{-}19$	516	$\mapsto$	250, 516
	$R{-}16$	238,	$\mapsto$	238, 245,
600	$R13$	[1980]	$\mapsto$	[1979]
601	$R21$	116	$\mapsto$	116,107
	$R22$	[1983] 107	$\mapsto$	[1984] 494
602	$R11$	149	$\mapsto$	150
	$L{-}17$	536	$\mapsto$	534
	$L{-}13$	75	$\mapsto$	
	$L{-}11$	O'Donell	$\mapsto$	O'Donnell
Index of Definitions				
607		leftmost 179	$\mapsto$	leftmost 180
608	$L12$		$\mapsto$	Paradox Curry- 573, 575 Liar's- 573
Index of Symbols				
613	11	λI-terms	$\mapsto$	$\lambda\mathsf{I}$-terms
614	8	$M(\!\mid\,\mid\!)$	$\mapsto$	$M(\!\mid\vec{N}\mid\!)$
	19	354	$\mapsto$	355
618		Add: $A =_\eta B$		trees A, B are equal up to (possibly infinitely many) η-conversions 240
619	-15	150	$\mapsto$	160
620 ($2\times$)	$11, 12$	$\Vert\!\models M = N$	$\mapsto$	$\models M = N$

Bibliography

Barendregt, H.P., M. Bunder and W. Dekkers [1993]. Systems of illative combinatory logic complete for first order propositional and predicate calculus, *The Journal of Symbolic Logic* **58**(3), pp. 89–108.

Folkerts, E. [1998]. Invertibility in lambda-eta, *Conference on Logic in Computer Science, LICS 1998*, IEEE, pp. 418–429.

Intrigila, B. and R. Statman [2004]. The omega rule is Π^0_2-hard in the $\lambda\beta$-calculus, *Conference on Logic in Computer Science, LICS 2004*, IEEE. To appear.

Jung, A. [1989]. *Cartesian closed categories of domains*, CWI Tract 66, Centrum voor Wiskunde en Informatica, Amsterdam.