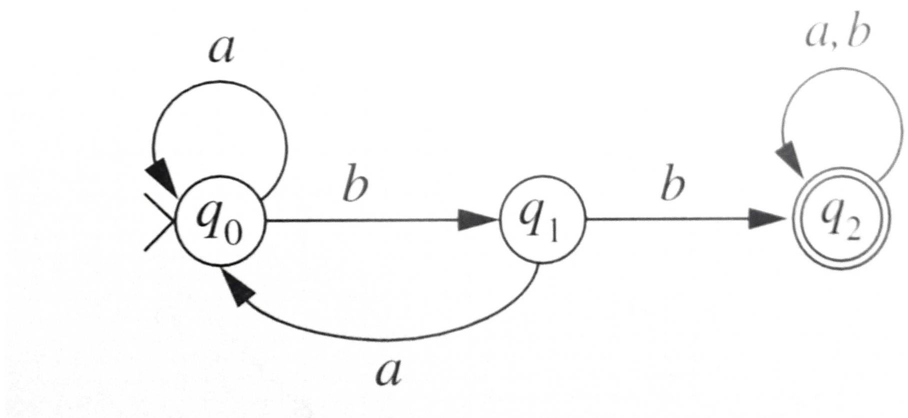
Finite Automata



Start indicated by “>”,
finish by double circle

$M = \langle Q, \Sigma, \delta, q_0, F \rangle$ with

$Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $F = \{q_2\}$ and δ given by

δ	q_0	q_1	$\boldsymbol{q_2}$
a	q_0	q_0	$\boldsymbol{q_2}$
b	q_1	$\boldsymbol{q_2}$	$\boldsymbol{q_2}$

Accepts $abba$, but not $baab$

M is a DFA over Σ if $M = (Q, \Sigma, q_0, \delta, F)$ with

Q	is a finite set of *'states'*
Σ	is a finite *alphabet*
$q_0 \in Q$	is the *initial* state
$F \subseteq Q$	is a finite set of *final* states
$\delta : Q \times \Sigma \to Q$	is the *transition* function (often given by a table)

Reading function $\hat{\delta} : Q \times \Sigma^* \to Q$ (arrival after multi-steps)

$$\begin{aligned} \hat{\delta}(q, \lambda) &= q \\ [\hat{\delta}(q, a) &= \delta(q, a)] \\ \hat{\delta}(q, wa) &= \delta(\hat{\delta}(q, w), a) \end{aligned}$$

Language accepted by M, notation $L(M)$:

$$L(M) = \{w \in \Sigma^* \mid \hat{\delta}(q_0, w) \in F\}$$

Computation for $\hat{\delta}(q, w)$ in the example $w = abba$:

$$
\begin{array}{lll}
[q, abba] & \vdash & [\delta(q, a), bba] \\
 & \vdash & [\delta(\delta(q, a), b), ba] \\
 & \vdash & [\delta(\delta(\delta(q, a), b), b), a] \\
 & \vdash & [\delta(\delta(\delta(\delta(q, a), b), b), a), \lambda]
\end{array}
\qquad
\begin{array}{l}
\hat{\delta}(q, a) = \delta(q, a) \\
\hat{\delta}(q, ab) = \delta(\delta(q, a), b) \\
\hat{\delta}(q, abb) = \delta(\delta(\delta(q, a), b), b) \\
\hat{\delta}(q, abba) = \delta(\delta(\delta(\delta(q, a), b), b), a)
\end{array}
$$

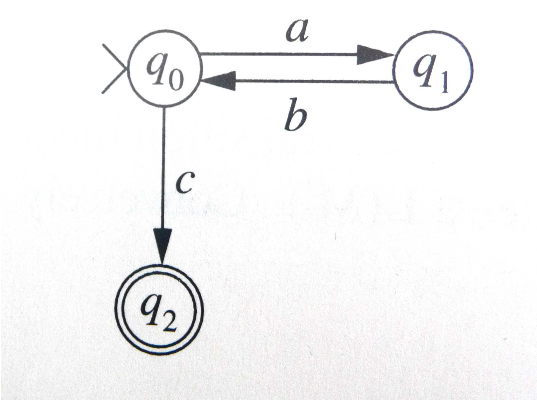
This computation corresponds to an equivalent definition of $\hat{\delta}$:

$$
\begin{array}{rcl}
\hat{\delta}(q, \lambda) & = & q \\
\hat{\delta}(q, a) & = & \delta(q, a) \\
\hat{\delta}(q, aw) & = & \hat{\delta}(\delta(q, a), w)
\end{array}
$$

Example transition table for δ with $Q = \{0, 1, 2, 3, 4\}$, $\Sigma = \{a, b\}$, $q_0 = 0$, and $F = \{4\}$

δ	a	b
0	1	0
1	1	2
2	1	3
3	4	0
4	4	4

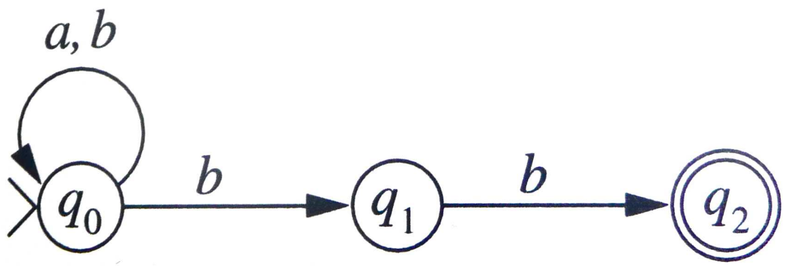
We have $\hat{\delta}(0, abba) = 4 \in F$ and $[0, abba] \vdash^* [4, \lambda]$, hence $abba \in L(M)$
Similarly $\hat{\delta}(0, baba) = 1 \notin F$; so even if $[0, baba] \vdash^* [1, \lambda]$ we have $baba \notin L(M)$.
Even if $\hat{\delta}(1, bba) = 4 \in F$ and $[1, bba] \vdash^* [4, \lambda]$ we have $bba \notin L(M)$.



δ	q_0	q_1	$\boldsymbol{q}_2$
a	q_1		
b		q_0	
c	$\boldsymbol{q}_2$		

stands for

δ	q_0	q_1	$\boldsymbol{q}_2$	q_e
a	q_1	q_e	q_e	q_e
b	q_e	q_0	q_e	q_e
c	$\boldsymbol{q}_2$	q_e	q_e	q_e



δ	q_0	q_1	$\boldsymbol{q}_2$
a	q_0	$\emptyset$	$\emptyset$
b	$\{q_0, q_1\}$	$\boldsymbol{q}_2$	$\emptyset$

in shorthand

δ	q_0	q_1	$\boldsymbol{q}_2$
a	q_0		
b	q_0, q_1	$\boldsymbol{q}_2$	

Prop. If a language L over Σ is accepted by a DFA,

then also $\overline{L} = \Sigma^* - L$.

Proof. Let L be accepted by $M = \langle Q, \Sigma, \delta, q_0, F \rangle$.

Then $\overline{L}$ is accepted by $M = \langle Q, \Sigma, \delta, q_0, \overline{F} \rangle$. ■

Prop. If L_1, L_2 are accepted by some NFA, then also $L_1 \cup L_2$.

Proof. "Put the two q_0-s together." ■

DFA	Deterministic finite automata
PFA	Partial deterministic finite automata
NFA	Non-deterministic finite automata

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$\delta : Q \times \Sigma \to Q$	is the *transition* function

DFA	$\delta : Q \times \Sigma \to Q$	given $q \in Q$ and $a \in \Sigma$, then $\delta(q, a) \in Q$
PFA	$\delta : Q \times \Sigma \hookrightarrow Q$	δ is *partial*: $\delta(q, a)$ is not always defined
NFA	$\delta : Q \times \Sigma \to \mathcal{P}(Q)$	$\delta(q, a)$ is *multiply defined*

We have DFA $\hookrightarrow$ PFA $\hookrightarrow$ NFA $\rightsquigarrow$ DFA

Officially a DFA is not an NFA:

the transition functions δ have different targets: Q resp. $\mathcal{P}(Q)$

But morally a DFA is an NFA: the uniquely determined $\delta(q, a) = q' \in Q$

can be considered as $\{q'\} \in \mathcal{P}(Q)$

So we can promote δ to $\overline{\delta}$ as follows: $\overline{\delta}(q, a) = \{\delta(q, a)\}$

giving the embedding DFA $\hookrightarrow$ NFA via

$$(Q, \Sigma, q_0, \delta, F) \ \rightsquigarrow \ (Q, \Sigma, q_0, \overline{\delta}, F)$$

From DFA to PFA there is a plain inclusion DFA $\subseteq$ PFA:

indeed every function is a partial function that happens to be total

The transition NFA $\rightsquigarrow$ DFA is via a modification of machines