Exercises

Formal languages, grammars, and automata

April 19, 2012

1. Regular languages

Note. Exercise 1.4 is only for devotees, to show that the subject is non-trivial.

For a $w \in \Sigma^*$, let $\#(w)$ be the number of symbols in w; moreover for $s \in \Sigma$, let $\#_s(w)$ be the number of occurrences of s in w. For example $\#(aab) = 3$, $\#_a(aab) = 2$, and $\#_b(aab) = 1$.

1.1. What are $L_1 = L((a \cup b)^*)$, $L_2 = L((a^*b^*)^*)$, and $L_3 = ((ab^*)^*)$. Show that precisely two of these languages are equal.

1.2. (i) Give a regular expression for

$$\{w \in \{a, b, c\}^* \mid \#(w) = 3\}.$$

(ii) Same for

$$\{w \in \{a, b, c\}^* \mid \#(w) \geq 3\}.$$

(iii) Same for

$$\{w \in \{a, b\}^* \mid aa \text{ occurs exactly twice in } w\}.$$

[Hint. Beware of the string aaa!]

1.3. Prove that

$$\{w \in \{a, b\}^* \mid bb \text{ does not occur in } w\} = L(a^*(baa^*)^*b?),$$

where $b? = (b \cup \lambda)$. We have omitted some parentheses; the full regular expression is $((a^*)((((ba)(a^*))^*)(b \cup \lambda)))$.

1.4*. [This exercise is at the moment rather hard, later less so!]
Show that the language

$$\{w \in \{a, b\}^* \mid \#_a(w) \text{ and } \#_b(w) \text{ are even}\}$$

is regular.

Easier is to show this for

$$\{w \in \{a, b\}^* \mid \#_a(w) \text{ is even}\}.$$